Value-at-risk in portfolio optimization: properties and computational approach

Alexei A. Gaivoronski
Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Alfred Getz vei 1, N-7049 Trondheim, Norway

Georg Pflug
Department of Statistics, University of Vienna, Universitätsstraße 5, A-1010 Vienna, Austria

Value-at-risk (VAR) is an important and widely used measure of the extent to which a given portfolio is subject to the risk present in financial markets. In this paper, we present a method of calculating a portfolio that gives the optimal VAR among those which yield at least some specified expected return. This method allows us to calculate the mean–VAR efficient frontier. The method is based on the approximation of historical VAR by smoothed VAR (SVAR), which filters out local irregular behavior of the historical VAR function. Moreover, we compare VAR as a risk measure to other well-known measures of risk, such as conditional value-at-risk (CVAR) and the standard deviation.

We show that the resulting efficient frontiers are quite different. An investor who wants to control his or her VAR should not look at portfolios lying on other than the VAR efficient frontier, although the calculation of this frontier is algorithmically more complex than other frontiers. We support this conjecture by presenting the results of a large-scale experiment with a representative selection of stock and bond indices from developed and emerging markets that involved the computation of many thousand VAR-optimal portfolios.

1 Introduction

Value-at-risk (VAR) is an important measure of the exposure of a securities portfolio to the different kinds of risk inherent in financial markets. It has now become an important risk management tool in the financial industry (RiskMetrics, 1995) and is part of the regulatory mechanisms for the industry (Bank for International Settlements, 1996). A considerable amount of research has been dedicated recently to the development of risk management methods based on VAR (see Jorion (2001) and Duffie and Pan (1997) for surveys).

In this paper we focus on the application of the VAR concept in the context of optimal portfolio selection. This is a relatively novel application of VAR, as opposed to the utilization of VAR for risk measurement purposes. The main obstacle here is that VAR optimization is inherently more difficult than, for example,

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